

Physics 129n: Solutions + Problem Set 6

1. a) Perkins 4.3

$$\text{eq 4.30 says } \Gamma(V \rightarrow \ell^+ \bar{\ell}) = \frac{16\pi \alpha^2 Q^2}{M_V^2} |\Psi(0)|^2$$

$$\text{w.t.m } Q^2 = |2\alpha \rho_\lambda|^2$$

$$\text{using } |\Psi(0)|^2 \sim \frac{1}{V} \sim \frac{1}{\frac{4}{3}\pi(0.6\text{fm})^3}$$

$$\Gamma = \left(\frac{16\pi (\frac{1}{3\pi})^2}{\frac{4}{3}\pi (0.6\text{fm})^3} \right) \underbrace{(197 \text{ MeV fm})^3}_{= k_C} \frac{Q^2}{M_V^2} \text{ to get units right}$$

$$= 2.263 \times 10^6 \text{ MeV}^3 \cdot \frac{Q^2}{M_V^2}$$

$$\rho \quad Q^2 = 1/2 \quad M_V = 776 \text{ MeV}$$

$$\Gamma = .0188 \text{ MeV}$$

$$\omega \quad Q^2 = 1/8 \quad M_V = 783 \text{ MeV}$$

$$\Gamma = .00205 \text{ MeV}$$

$$\phi \quad Q^2 = 1/9 \quad M_V = 1019$$

$$\Gamma = .00242 \text{ MeV}$$

the ρ is off by a factor of 3, as is the ω

the ϕ is off by factor of 1.5

Not bad given how crude the approx is

$$b) \Gamma_{\psi \rightarrow ee} = 5.3 \text{ keV}$$

$$Q^2 = 4/9$$

IF we scaled so $\frac{|\Psi(0)|^2}{M_V^3}$ const we would predict

$$\Gamma_{\psi \rightarrow ee} = 4 \Gamma_{\phi \rightarrow ee}$$

using correct $\Gamma_{\phi \rightarrow ee} = 1.37 \text{ keV}$, predict

$$\Gamma_{\psi \rightarrow ee} = 5.48 \text{ keV} \quad \underline{\text{right on}}$$

also predict $\Gamma_{Y \rightarrow ee} = \Gamma_{\phi \rightarrow ee} = 1.37 \text{ keV}$

when true value is 1.32 keV. So $\frac{|\Psi(0)|^2}{M_V^3} = \text{const}$

2. Perkins 4.5

eq 4.8 says for triplet position

$$\Gamma(3\gamma) = \frac{2(\pi^2 - 9)}{9\pi} \alpha^6 m$$

replacing $\alpha \rightarrow \frac{4}{3}\alpha_s$

$$\begin{aligned} \Gamma(\psi \rightarrow 3g) &= \frac{2(\pi^2 - 9)}{9\pi} \alpha_s^6 \left(\frac{4}{3}\right)^6 m_\psi \\ &= (63456)(3100 \times 10^3 \text{ keV}) \alpha_s^6 \end{aligned}$$

$$\begin{aligned} \therefore \alpha_s^6 &= (1.04 \times 10^6 \text{ keV})^{-1} \Gamma(\psi \rightarrow 3g) \\ &= \frac{(87 \text{ keV})(.88)}{1.04 \times 10^6 \text{ keV}} = 71 \times 10^{-6} \end{aligned}$$

$$\alpha_s = 2 \times 10^{-1} = .2$$

3. Perkins 4.6

Note: the α_s^6 above comes from an α_s^3 due to 3g decay and a $|\psi(0)|^2$ which goes as $\frac{1}{\pi a^3}$ with $a \sim \frac{1}{\alpha_s}$

so, if we replace a g with a f we get

$$\frac{\Gamma_{Y \rightarrow 2g + f}}{\Gamma_{Y \rightarrow 3g}} = \frac{\alpha}{\frac{4}{3}\alpha_s} Q_{b\bar{b}}^2 = \frac{\alpha}{12\alpha_s}$$

$$\text{BR}(Y \rightarrow f + \text{hadrons}) = .3\%$$

$$\text{BR}(Y \rightarrow \text{hadrons}) = 92.5\%$$

we got the 2nd BR by noting that

$$\Gamma_{\gamma \rightarrow ee} = 1.3 \text{ keV} \quad \text{so}$$

$$\Gamma_{\gamma \rightarrow \text{hadrons}} = \Gamma_{\gamma \text{ tot}} - 3 \underbrace{\Gamma_{\gamma \rightarrow ee}}_{\text{3 lepton species}}$$

$$\begin{aligned} \Gamma_{\gamma \rightarrow \text{hadrons}} &= \Gamma_{\gamma \text{ tot}} - 3 \Gamma_{\gamma \rightarrow ee} \\ &= 53 \text{ keV} - 3.9 \text{ keV} \\ &= 49.1 \text{ keV} \end{aligned}$$

$$\text{BR} = \frac{49.1}{53} = 92.5\%$$

$$\therefore \alpha_s = \frac{\alpha}{12} \frac{92.5}{.3} = \\ = .188$$

Note: this does not agree with back of Perkins!

$$4. R = \frac{e^+ e^- \rightarrow \text{hadrons}}{e^+ e^- \rightarrow \mu^+ \mu^-} = \sum_{\substack{\text{color} \\ \text{3}}} e_q^2$$

R above $t\bar{t}$ threshold is :

$$3 \left(\frac{4}{a} + \frac{1}{a} + \frac{4}{a} + \frac{1}{a} + \frac{4}{a} + \frac{1}{a} \right) = 5$$